Sets and Union- Keynotes

A set can be defined as a collection of things that are brought together because they obey a certain rule. These 'things' may be anything you like: numbers, people, shapes, cities, bits of text ..., literally anything. The key fact about the 'rule' they all obey is that it must be well-defined. In other words, it enables us to say for sure whether or not a given 'thing' belongs to the collection. If the 'things' we're talking about are English words, for example, a well-defined rule might be: '... has 5 or more letters'. A rule which is not well-defined (and therefore couldn't be used to define a set) might be: '... is hard to spell'

Elements

A 'thing' that belongs to a given set is called an element of that set.
For example: Henry VIII is an element of the set of Kings of England

Notation

Curly brackets {...... } are used to stand for the phrase 'the set of ...'. These braces can be used in various ways.

For example: We may list the elements of a set: $\{-3, -2, -1, 0, 1, 2, 3\}$.

We may describe the elements of a set: { integers between − 3 and 3 inclusive}.

We may use an identifier (the letter x for example) to represent a typical element, a | symbol to stand for the phrase 'such that', and then the rule or rules that the identifier must obey:

$\{x \mid x \text{ is an integer and } |x| < 4\}$ or $\{x \mid x \in Z, |x| < 4\}$

The last way of writing a set - called set comprehension notation - can be generalized as:

x | P(x), where P(x) is a statement (technically a propositional function) about x and the set is the collection of all elements x for which P is true.

The symbol $\in$ is used as follows:

$\in$ means 'is an element of ...'. For example: dog $\in$ {quadrupeds}

□ means 'is not an element of ...'. For example:

Washigton DC □ {European capital cities}

A set can be finite: {British citizens}or infinite: {7, 14, 21, 28, 35, …. }.

Sets will usually be denoted using upper case letters: A, B, ...

Elements will usually be denoted using lower case letters: x, y, ...

Some Special Sets

1. Universal Set

The set of all the 'things' currently under discussion is called the universal set (or sometimes, simply the universe). It is denoted by U. The universal set doesn't contain everything in the whole universe. On the contrary, it restricts us to just those things that are relevant at a particular time. For example, if in a given situation we're talking about numeric values – quantities, sizes, times, weights, or whatever – the universal set will be a suitable set of numbers (see below). In another context, the universal set may be {alphabetic characters} or {all living people}, etc.

2. Empty set

The set containing no elements at all is called the null set, or empty set. It is denoted by a pair of empty braces: { } or by the symbol f. It may seem odd to define a set that contains no elements. Bear in mind, however, that one may be looking for solutions to a problem where it isn't clear at the outset whether or not such solutions even exist. If it turns out that there isn't a solution, then the set of solutions is empty.

For example:

If U = {words in the English language} then {words with more than 50 letters}= f .

If U = {whole numbers} then {x|x2 = 10} = f .

Operations on the empty set

Operations performed on the empty set (as a set of things to be operated upon) can also be confusing. (Such operations are nullary operations.) For example, the sum of the elements of the empty set is zero, but the product of the elements of the empty set is one (see empty product). This may seem odd, since there are no elements of the empty set, so how could it matter whether they are added or multiplied (since "they" do not exist)? Ultimately, the results of these operations say more about the operation in question than about the empty set. For instance, notice that zero is the identity element for addition, and one is the identity element for multiplication.

3. Equality

Two sets A and B are said to be equal if and only if they have exactly the same elements. In this case, we simply write:

A = B

Note two further facts about equal sets:

The order in which elements are listed does not matter.

If an element is listed more than once, any repeat occurrences are ignored.

So, for example, the following sets are all equal:

{1, 2, 3} = {3, 2, 1} = {1, 1, 2, 3, 2, 2}

(You may wonder why one would ever come to write a set like {1, 1, 2, 3, 2, 2}. You may recall that when we defined the empty set we noted that there may be no solutions to a particular problem - hence the need for an empty set. Well, here we may be trying several different approaches to solving a problem, some of which in fact lead us to the same solution. When we come to consider the distinct solutions, however, any such repetitions would be ignored.)

4. Subsets

If all the elements of a set A are also elements of a set B, then we say that A is a subset of B, and we write: $A \subseteq B$

For example: If T = {2, 4, 6, 8, 10} and E = {even integers}, then $T \subseteq E$

If A = {alphanumeric characters} and P = {printable characters}, then $A \subseteq P$

If Q = {quadrilaterals} and F = {plane figures bounded by four straight lines}, then $Q \subseteq F$

Notice that $A \subseteq B$ does not imply that B must necessarily contain extra elements that are not in A; the two sets could be equal – as indeed Q and F are above. However, if, in addition, B does contain at least one element that isn't in A, then we say that A is a proper subset of B. In such a case we would write: $A \subset B$

In the examples above:

E contains 12, 14, ... , so $T \subset E$

P contains $, ;, &, ..., so $A \subset P$

But Q and F are just different ways of saying the same thing, so Q = F.

The use of $\subset$ and $\subseteq$ is clearly analogous to the use of $<$ and $\leq$ when comparing two numbers.

Note: Every set is a subset of the universal set, and the empty set is a subset of every set.

5. Disjoint

Two sets are said to be disjoint if they have no elements in common.

For example: If A = {even numbers} and B = {1, 3, 5, 11, 19}, then A and B are disjoint.

Operations on Sets

1. Intersection

The intersection of two sets A and B, written $A \cap B$, is the set of elements that are in A and in B.

(Note that in symbolic logic, a similar symbol,^, is used to connect two logical propositions with the AND operator.)

For example, if A = {1, 2, 3, 4} and B = {2, 4, 6, 8}, then $A \cap B$ = {2, 4}.

We can say, then, that we have combined two sets to form a third set using the operation of intersection.

2. Union

In a similar way we can define the union of two sets as follows:

The union of two sets A and B, written $A \cup B$, is the set of elements that are in A or in B (or both).

(Again, in logic a similar symbol,V, is used to connect two propositions with the OR operator.)

So, for example, {1, 2, 3, 4} ∪ {2, 4, 6, 8} = {1, 2, 3, 4, 6, 8}.

You'll see, then, that in order to get into the intersection, an element must answer 'Yes' to both questions, whereas to get into the union, either answer may be 'Yes'.

The ∪ symbol looks like the first letter of 'Union' and like a cup that will hold a lot of items. The ∩ symbol looks like a spilled cup that won't hold a lot of items, or possibly the letter 'n', for intersection. Take care not to confuse the two.

3. Difference

The difference of two sets A and B (also known as the set-theoretic difference of A and B, or the relative complement of B in A) is the set of elements that are in A but not in B.

This is written A - B, or sometimes A \ B.

For example, if A = {1, 2, 3, 4} and B = {2, 4, 6, 8}, then A - B = {1, 3}.

4. Complement

The set of elements that are not in a set A is called the complement of A. It is written A′ (or sometimes AC, or Â). Clearly, this is the set of elements that answer 'No' to the question Are you in A?.

For example, if U = N and A = {odd numbers}, then A′ = {even numbers}.

Notice the spelling of the word complement: its literal meaning is 'a complementary item or items'; in other words, 'that which completes'. So if we already have the elements of A, the complement of A is the set that completes the universal set.

5. Cardinality

The cardinality of a finite set A, written | A | (sometimes #(A) or n(A)), is the number of (distinct) elements in A. So, for example:

If A = {lower case letters of the alphabet}, | A | = 26.

Some special sets of numbers

Several sets are used so often, they are given special symbols.

1. The natural numbers

The 'counting' numbers (or whole numbers) starting at 1, are called the natural numbers. This set is sometimes denoted by N. So N = {0, 1, 2, 3, ...}.

Note that, when we write this set by hand, we can't write in bold type so we write an N in blackboard bold font: N

2. Integers

All whole numbers, positive, negative and zero form the set of integers. It is sometimes denoted by Z. So Z = {..., -3, -2, -1, 0, 1, 2, 3, ...}

In blackboard bold, it looks like this: Z

3. Real numbers

If we expand the set of integers to include all decimal numbers, we form the set of real numbers. The set of reals is sometimes denoted by R.

A real number may have a finite number of digits after the decimal point (e.g. 3.625), or an infinite number of decimal digits. In the case of an infinite number of digits, these digits may:

recur; e.g. 8.127127127...

... or they may not recur; e.g. 3.141592653...

In blackboard bold: R

4. Rational numbers

Those real numbers whose decimal digits are finite in number, or which recur, are called rational numbers. The set of rationals is sometimes denoted by the letter Q.

A rational number can always be written as exact fraction p/q; where p and q are integers. If q equals 1, the fraction is just the integer p. Note that q may NOT equal zero as the value is then undefined.

For example: 0.5, -17, 2/17, 82.01, 3.282828... are all rational numbers.

In blackboard bold: Q

5. Irrational numbers

If a number can't be represented exactly by a fraction p/q, it is said to be irrational.

Examples include: √2, √3.

Concepts and Theory

For a group of two sets:

1. If A and B are overlapping set, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

2. If A and B are disjoint set, $n(A \cup B) = n(A) + n(B)$

3. $n(A) = n(A \cup B) + n(A \cap B) - n(B)$

4. $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

5. $n(B) = n(A \cup B) + n(A \cap B) - n(A)$

6. $n(U) = n(A) + n(B) - n(A \cap B) + n((A \cup B)^c)$

7. $n((A \cup B)^c) = n(U) + n(A \cap B) - n(A) - n(B)$

8. $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

9. $n(A - B) = n(A \cup B) - n(B)$

10. $n(A - B) = n(A) - n(A \cap B)$

11. $n(A^c) = n(U) - n(A)$

For a group of three sets:

1. $n(A \cup B \cup C) = n(U) - n((A \cup B \cup C)^c)$

2. $n_0(A) = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

3. $n_0(B) = n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

4. $n_0(C) = n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

5. $n(A \cap B only) = n(A \cap B) - n(A \cap B \cap C)$

6. $n(B \cap C only) = n(B \cap C) - n(A \cap B \cap C)$

7. $n(A \cap C only) = n(A \cap C) - n(A \cap B \cap C)$

Exercise Questions

1.Which of the following set is equivalent to set A= {a,b,c,d,e}

a. B= {1,2,3,4,5} b. B= {c,a,b,f} c. B= {-1,0,2,4} d. None of these

2.If A and B are two sets, then (A-B)UB is…………………..

a. A b. B c. AUB d. A∩B

3.If A and B are two sets, then (A-B) ∩ B is……………………….

a. A b. B c. A∩B d.{ }

4.If A⊂B⊂C, then (A-B)U(B-C)U(A-C)=…….

a. A∩B∩C b. AUBUC c. { } d. None of these

5.Find the solution set of the equation $x^2+x+2=0$ in roster form

a. {1,-2} b. { } c. {1,1} d. {1}

6.Find the roster form of the following set

E= The set of all letters in the word TRIGNOMETRY

a. E= {T,R,I,G,N,O,M,E,T,R,Y} b. E= { } c. E={T,R,I,G,N,O,M,E,Y} d. None of these

7.Convert the A= {3,-3} into set-builder form

a. A= {x:x is a positive integer and is a divisor of 19}

b. A= {x:x is an integer and $x^2-9=0$}

c. A= {x:x is an integer and x+1=1}

d. None of these

8.Find the number of elements in the power set of {1,2}

a. 4 b. 0 c. 2 d. None of these

9.B={x:x is an even natural number less than 6}Ì A= {x:x is a natural number which divides 36}. Find B in roster form

a. B= {2,3,4,6} b. B= {2} c. { } d. None of these

10.Number of subsets of A= {0}

a. 1 b. 0 c. 2 d. None of these

11. In a class of 120 students numbered 1 to 120, all even numbered students opt for Physics, whose numbers are divisible by 5 opt for Chemistry and those whose numbers are divisible by 7 opt for Math. How many opt for none of the three subjects?

a.19 b.41 c.21 d.26

12. Of the 200 candidates who were interviewed for a position at a call center, 100 had a two-wheeler, 70 had a credit card and 140 had a mobile phone. 40 of them had both, a two-wheeler and a credit card, 30 had both, a credit card and a mobile phone and 60 had both, a two wheeler and mobile phone and 10 had all three. How many candidates had none of the three?

a. 0 b. 20 c. 10 d.18

13. In a class of 40 students, 12 enrolled for both English and German. 22 enrolled for German. If the students of the class enrolled for at least one of the two subjects, then how many students enrolled for only English and not German?
a.30 b. 10 c. 18 d.8

14. In a class 40% of the students enrolled for Math and 70% enrolled for Economics. If 15% of the students enrolled for both Math and Economics, what % of the students of the class did not enroll for either of the two subjects?

a.5% b.15% c. 0% d. 25%

Answer & Explanations

1. Option (a)

Number of elements of A= Number of elements of B

2.c

3.d

4.c

5. Option (a)

(x+2)(x-1)=0

x=-2 or 1

6. Option (c)

Avoid repeated letters

7.b

8. Option (a)

Power set is the set of subsets of {1,2}

That is { {1,2}, {1}, {2}, f}

9.b

10.c

11. Number of students who took at least one of the three subjects can be found by finding out A U B U C, where A is the set of those who took Physics, B the set of those who took Chemistry and C the set of those who opted for Math.

Now, AUBUC = A + B + C - (A n B + B n C + C n A) + (A n B n C)
A is the set of those who opted for Physics = 120/2 = 60 students
B is the set of those who opted for Chemistry = 120/5 = 24
C is the set of those who opted for Math = 120/7 = 17.
The 10th, 20th, 30th…… numbered students would have opted for both Physics and Chemistry.
Therefore, A n B = 120/10 = 12
The 14th, 28th, 42nd…. Numbered students would have opted for Physics and Math.
Therefore, C n A = 120/14 = 8
The 35th, 70th …. Numbered students would have opted for Chemistry and Math.
Therefore, A n B = 120/35 = 3
And the 70th numbered student would have opted for all three subjects.
Therefore, AUBUC = 60 + 24 + 17 - (12 + 8 + 3) + 1 = 79.
Number of students who opted for none of the three subjects = 120 - 79 = 41.

12. Number of candidates who had none of the three = Total number of candidates - number of candidates who had at least one of three devices.

Total number of candidates = 200.
Number of candidates who had at least one of the three = A U B U C, where A is the set of those who have a two wheeler, B the set of those who have a credit card and C the set of those who have a mobile phone.
We know that AUBUC = A + B + C - {A n B + B n C + C n A} + A n B n C
Therefore, AUBUC = 100 + 70 + 140 - {40 + 30 + 60} + 10

Or AUBUC = 190.
As 190 candidates who attended the interview had at least one of the three gadgets, 200 - 190 = 10 candidates had none of three.

13. The correct choice is (C) and the correct answer is 18.

Let A be the set of students who have enrolled for English and B be the set of students who have enrolled for German.
Then, (A U B) is the set of students who have enrolled at least one of the two subjects. As the students of the class have enrolled for at least one of the two subjects, A U B = 40
We know A U B = A + B - (A n B)
i.e, 40 = A + 22 - 12
or A = 30 which is the set of students who have enrolled for English and includes those who have enrolled for both the subjects.
However, we need to find out the number of students who have enrolled for only English = Students enrolled for English - Students enrolled for both German and English= 30 - 12 = 18.

14. The correct choice is (A) and the correct answer is 5%.

We know that (A U B) = A + B - (A n B), where (A U B) represents the set of people who have enrolled for at least one of the two subjects Math or Economics and (A n B) represents the set of people who have enrolled for both the subjects Math and Economics.
Note: (A U B) = A + B - (A n B) => (A U B) = 40 + 70 - 15 = 95%
That is 95% of the students have enrolled for at least one of the two subjects Math or Economics.
Therefore, the balance (100 - 95)% = 5% of the students have not enrolled for either of the two subjects.